Exercise 50

Calculate y'.

$$y = \sin^2\left(\cos\sqrt{\sin \pi x}\right)$$

Solution

Calculate y' by using the chain rule repeatedly.

$$\begin{aligned}
y' &= \frac{d}{dx}\sin^2\left(\cos\sqrt{\sin \pi x}\right) \\
&= \frac{d}{dx}\left[\sin\left(\cos\sqrt{\sin \pi x}\right)\right]^2 \\
&= 2\left[\sin\left(\cos\sqrt{\sin \pi x}\right)\right]^1 \cdot \frac{d}{dx}\sin\left(\cos\sqrt{\sin \pi x}\right) \\
&= 2\sin\left(\cos\sqrt{\sin \pi x}\right) \cdot \cos\left(\cos\sqrt{\sin \pi x}\right) \cdot \frac{d}{dx}\left(\cos\sqrt{\sin \pi x}\right) \\
&= 2\sin\left(\cos\sqrt{\sin \pi x}\right) \cdot \cos\left(\cos\sqrt{\sin \pi x}\right) \cdot \left(-\sin\sqrt{\sin \pi x}\right) \cdot \frac{d}{dx}\sqrt{\sin \pi x} \\
&= 2\sin\left(\cos\sqrt{\sin \pi x}\right) \cdot \cos\left(\cos\sqrt{\sin \pi x}\right) \cdot \left(-\sin\sqrt{\sin \pi x}\right) \cdot \frac{1}{2}(\sin \pi x)^{-1/2} \cdot \frac{d}{dx}(\sin \pi x) \\
&= 2\sin\left(\cos\sqrt{\sin \pi x}\right) \cdot \cos\left(\cos\sqrt{\sin \pi x}\right) \cdot \left(-\sin\sqrt{\sin \pi x}\right) \cdot \frac{1}{2}(\sin \pi x)^{-1/2} \cdot (\cos \pi x) \cdot \frac{d}{dx}(\pi x) \\
&= \left[2\sin\left(\cos\sqrt{\sin \pi x}\right)\cos\left(\cos\sqrt{\sin \pi x}\right)\right] \cdot \left(-\sin\sqrt{\sin \pi x}\right) \cdot \frac{1}{2}(\sin \pi x)^{-1/2} \cdot (\cos \pi x) \cdot (\pi) \\
&= \sin\left(2\cos\sqrt{\sin \pi x}\right) \cdot \left(-\sin\sqrt{\sin \pi x}\right) \cdot \frac{1}{2\sqrt{\sin \pi x}} \cdot (\pi \cos \pi x) \\
&= -\frac{\pi \sin\left(2\cos\sqrt{\sin \pi x}\right)\sin\left(\sqrt{\sin \pi x}\right)\cos \pi x}{2\sqrt{\sin \pi x}}
\end{aligned}$$